

Circle Coordinate Geometry 1

1.

In each case, show that the circle passes through the given point:

a $(x - 2)^2 + (y - 5)^2 = 13$, point (4, 8)

b $(x + 7)^2 + (y - 2)^2 = 65$, point (0, -2)

c $x^2 + y^2 = 25^2$, point (7, -24)

d $(x - 2a)^2 + (y + 5a)^2 = 20a^2$, point (6a, -3a)

e $(x - 3\sqrt{5})^2 + (y - \sqrt{5})^2 = (2\sqrt{10})^2$ point, $(\sqrt{5}, -\sqrt{5})$

2.

The point (4, -2) lies on the circle centre (8, 1).
Find the equation of the circle.

Hint

First find the radius of the circle.

3.

The line PQ is the diameter of the circle, where P and Q are (5, 6) and (-2, 2) respectively.
Find the equation of the circle. (5 marks)

4.

The point (1, -3) lies on the circle $(x - 3)^2 + (y + 4)^2 = r^2$. Find the value of r . (3 marks)

5.

a Show that $x^2 + y^2 - 10x + 4y - 20 = 0$ can be written in the form $(x - a)^2 + (y - b)^2 = r^2$,
where a , b and r are numbers to be found. (2 marks)

b Hence write down the centre and radius of the circle with equation
 $x^2 + y^2 - 10x + 4y - 20 = 0$. (2 marks)

6.

Find the centre and radius of the circle with each of the following equations.

a $x^2 + y^2 - 2x + 8y - 8 = 0$

b $x^2 + y^2 + 12x - 4y = 9$

c $x^2 + y^2 - 6y = 22x - 40$

d $x^2 + y^2 + 5x - y + 4 = 2y + 8$

e $2x^2 + 2y^2 - 6x + 5y = 2x - 3y - 3$

Hint

Start by writing the equation
in one of the following forms:

$$(x - a)^2 + (y - b)^2 = r^2$$

$$x^2 + y^2 + 2fx + 2gy + c = 0$$

7.

A circle C has equation $x^2 + y^2 + 12x + 2y = k$, where k is a constant.

a Find the coordinates of the centre of C .

(2 marks)

b State the range of possible values of k .

(2 marks)

Problem-solving

A circle must have a
positive radius.

8.

The circle with equation $(x - k)^2 + y^2 = 41$ passes through the point (3, 4).
Find the two possible values of k . (5 marks)

9.

The line $y = x + 4$ meets the circle $(x - 3)^2 + (y - 5)^2 = 34$ at A and B .
Find the coordinates of A and B .

10.

Show that the line $x - y - 10 = 0$ does not meet the circle $x^2 - 4x + y^2 = 21$.

Problem-solving

Attempt to solve the equations simultaneously. Use the discriminant to show that the resulting quadratic equation has no solutions.

11.

- a** Show that the line $x + y = 11$ meets the circle with equation $x^2 + (y - 3)^2 = 32$ at only one point. **(4 marks)**
- b** Find the coordinates of the point of intersection. **(1 mark)**

12.

The line $y = 2x - 2$ meets the circle $(x - 2)^2 + (y - 2)^2 = 20$ at A and B .

- a** Find the coordinates of A and B . **(5 marks)**
- b** Show that AB is a diameter of the circle. **(2 marks)**

13.

The line with equation $y = kx$ intersects the circle with equation $x^2 - 10x + y^2 - 12y + 57 = 0$ at two distinct points.

- a** Show that $21k^2 - 60k + 32 < 0$. **(5 marks)**
- b** Determine the range of possible values for k . Round your answer to 2 decimal places. **(3 marks)**

14.

The line with equation $y = 4x - 1$ does not intersect the circle with equation $x^2 + 2x + y^2 = k$. Find the range of possible values of k .

15.

The line with equation $y = 2x + 5$ meets the circle with equation $x^2 + kx + y^2 = 4$ at exactly one point. Find two possible values of k .